Name: SOLUTION ID#: ______________________

LAU SCHOOL OF ENGINEERING
Lebanese American University
Department of Electrical and Computer Engineering

ELE 302 – Electrical Circuits II
Midterm Exam I – Fall 2013

Duration: 1 hour 30 mins
Start Time: 7:00 pm

Date: 06/11/2013
Dr. Dani Tannir

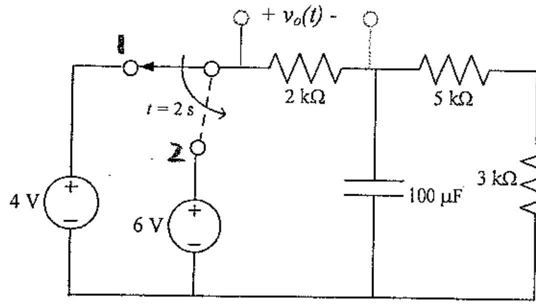
INSTRUCTIONS:

- Answer each of the following questions in the space provided.
- This is a closed-book exam.
- If something is not clear, state your assumptions.
- Programmable calculators are not allowed.
- The number of marks for each question is specified next to it.
- The total number of marks is 100.

1	2	3	4	Total
20 /20	20 /20	30 /30	30 /30	100

Question 1 [20 marks]

In the following circuit, the switch was in position 1 for a long time before switching to position 2 at t = 2 s

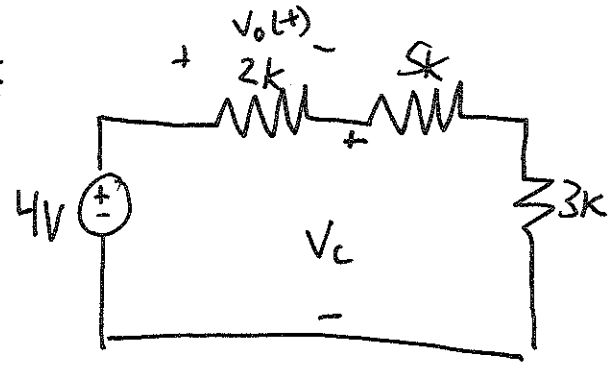


(4) a. Determine $v_o(t)$ $t < 2$ s

(12) b. Determine $v_o(t)$ $t > 2$ s

(4) c. Sketch $v_o(t)$ for $t > 0$

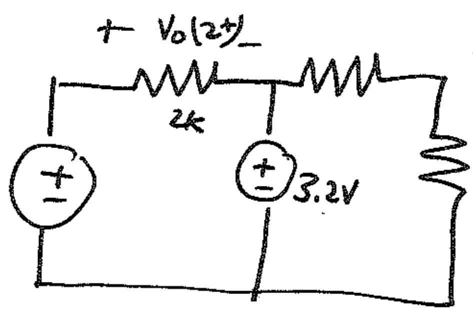
a) $t < 2$



$V_o = \frac{2}{10} 4 = 0.8$

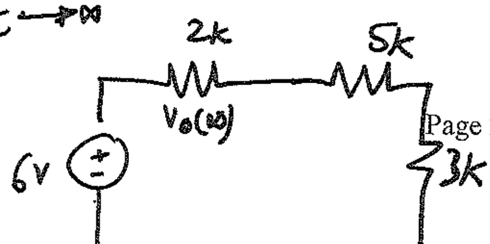
b) $t = 2^-$ $V_c(2^-) = 3.2V$

$t = 2^+$



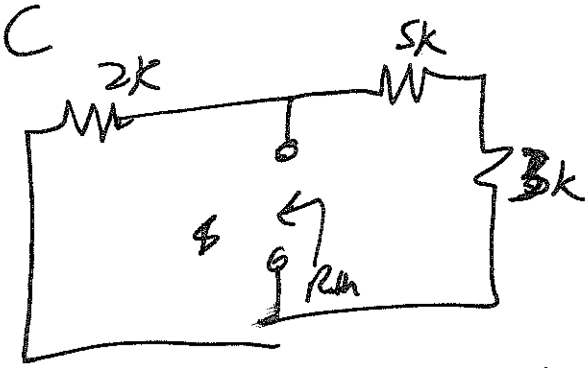
$V_o(2^+) = 6 - 3.2 = 2.8V$

$t \to \infty$



$V_o(\infty) = \frac{2}{10}(6)$
$= 1.2V$

$\tau = R_{th} C$



$R_{th} = 2k \| (3k + 5k)$

$= 1.6K$

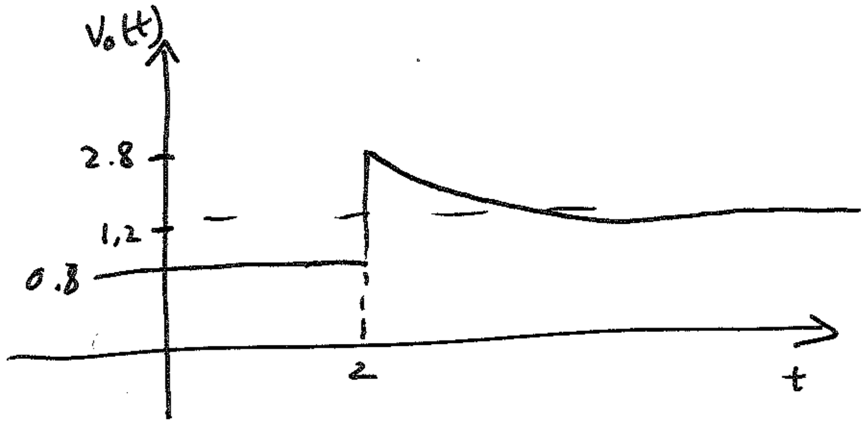
$\tau = (1.6k)(0.1m) = 0.16s$

$V_o(t) = V_o(\infty) + [V_o(2+) - V_o(\infty)] e^{-(t-2)/\tau}$

$= 1.2 + [2.8 - 1.2] e^{-(t-2)/0.16}$

$= 1.2 + 1.6 e^{-(t-2)/0.16}$

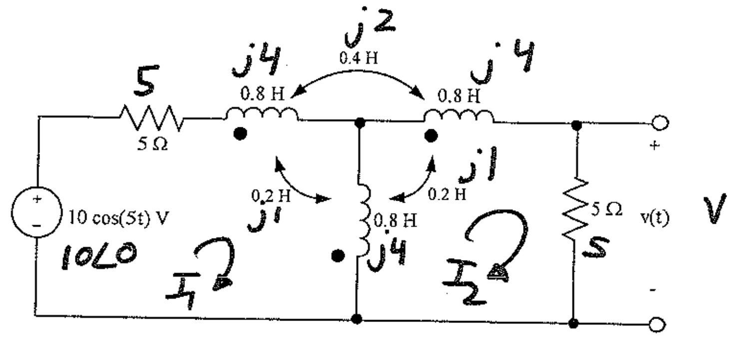
c)



Question 2 [20 marks]

Determine the output voltage $v(t)$ in the following circuit.

<u>Hint:</u> Use phasors



Mesh ①

$$10\angle 0 = 5I_1 + j4I_1 + j2I_2 - j1(I_1 - I_2) + j4(I_1 - I_2) - j1I_1 - j1I_2$$

$$10\angle 0 = (5 + j4 - j1 + j4 - j1)I_1 + (j2 + j1 - j4 - j1)I_2$$

$$\Rightarrow \boxed{10\angle 0 = (5 + j6)I_1 + (-j2)I_2} \quad (1)$$

Mesh ②

$$5I_2 + j4(I_2 - I_1) + j1I_2 + j1I_1 + j4I_2 + j2I_1 + j1(I_2 - I_1) =$$

$$(-j4 + j1 + j2 - j1)I_1 + (5 + j4 + j1 + j4 + j1)I_2 = 0$$

$$\boxed{-j2I_1 + (5 + j10)I_2 = 0} \quad (2)$$

$$I_1 = \frac{5 + j10}{j2} I_2 = (5 - j2.5)I_2$$

$$\Rightarrow 10\angle 0 = (40 + j17.5)I_2 - j2I_2$$

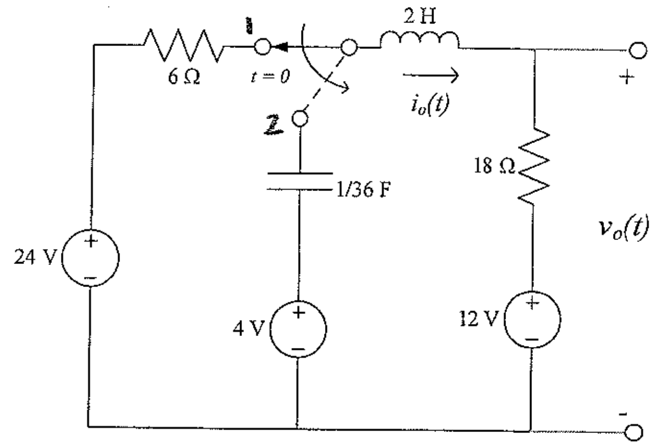
$$\Rightarrow I_2 = 0.23\angle -21.2^\circ$$

$$\Rightarrow V = 1.165\angle -21.2^\circ$$

$$\Rightarrow v(t) = 1.165 \cos(5t - 21.2^\circ) ✓$$

Question 3 [30 marks]

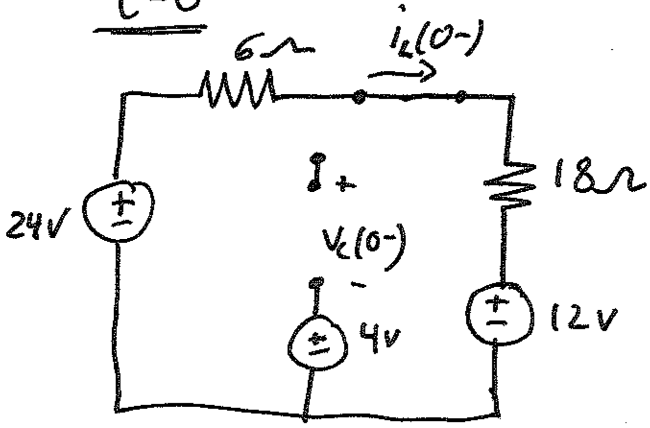
The switch in the given network was in position 1 for a long time before moving to position 2 at t = 0.



a) $i_L(0-) \rightarrow$ (3)
$V_C(0-) \rightarrow$ (3)
D.E. $\rightarrow$ (6)
Roots $\rightarrow$ (3)
expression $\rightarrow i_o(t) \rightarrow$ (2)
$\rightarrow V_o(t) \rightarrow$ (3)
$K_1, K_2, K_3 \rightarrow$ (6)
b) (4)

a. Find $v_o(t)$ for $t > 0$
b. Determine how much energy is stored in the inductor at t = 50ms?

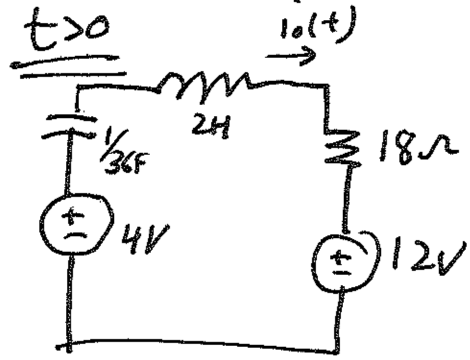
$t=0^-$



$$i_L(0-) = \frac{24-12}{24} = 0.5A$$

$$V_c(0-) = 0V$$

$t>0$



$$Ri_o + L\frac{di_o}{dt} + \frac{1}{C}\int i_o(t)dt = -8$$

$$18i_o + 2\frac{di_o}{dt} + 36\int i_o dt = -8$$

$$18\frac{di_o}{dt} + 2\frac{d^2 i_o}{dt^2} + 36 i_o = 0$$

$$\frac{d^2 i_o}{dt^2} + 9\frac{di_o}{dt} + 18 i_o = 0$$

$$s^2 + 9s + 18$$

$s_1 = -3$
$s_2 = -6$ $\Rightarrow$ Real unequal roots overdamped

$$i_o(t) = K_1 e^{-3t} + K_2 e^{-6t} + K_3$$

$$K_3 = i_o(\infty) = 0$$

$$i_o(0+) = K_1 + K_2 \qquad \frac{di_o(0+)}{dt} = -3K_1 - 6K_2$$

$$0.5 = K_1 + K_2$$

$$\frac{di(0+)}{dt} = \frac{-8 - V_c(0+) - 18 i_o(0+)}{2}$$

$$= \frac{-8 - 0 - 9}{2} = -\frac{17}{2}$$

$$0.5 = K_1 + K_2$$

$$-\frac{17}{2} = -3K_1 - 6K_2$$

$$K_1 = -1.833 \quad \left(-\tfrac{11}{6}\right)$$

$$K_2 = 2.333 \quad \left(\tfrac{14}{6}\right)$$

$$i_o(t) = -\frac{11}{6} e^{-3t} + \frac{14}{6} e^{-6t} \text{ A}$$

$$V_o(t) = 12 + 18 i_o(t) = 12 - 33e^{-3t} + 42e^{-6t}$$

b) $E = \frac{1}{2} L i_o^2(t)$

$$i_o(50m) = -1.577 + 1.729$$

$$= 0.151$$

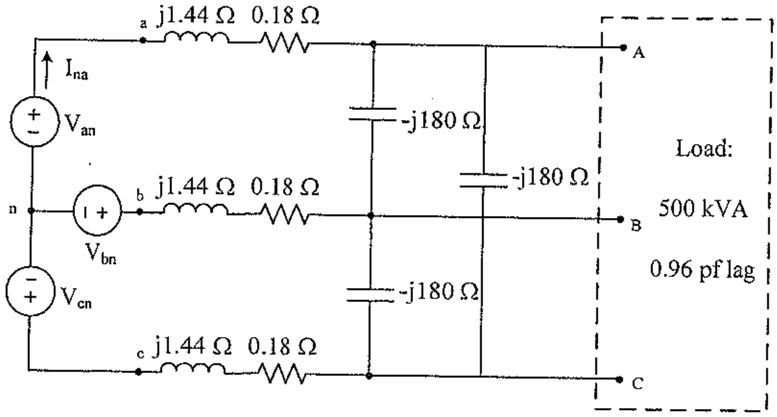
$$\boxed{E = 0.0228 \text{ J}}$$

Question 4 [30 marks]

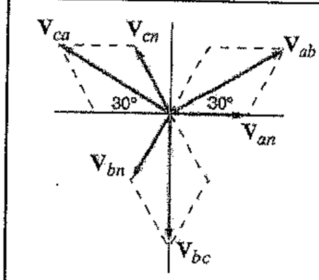
The magnitude of the line-to-neutral voltage at the terminals of the balanced 3-phase load in the circuit shown below is **1200V**. At this voltage, the load is absorbing 500 kVA at 0.96 pf lag.

(20) a. Use $\mathbf{V}_{AN}$ as the reference (i.e. phase angle of $\mathbf{V}_{AN} = 0$) to compute the current $\mathbf{I}_{na}$ as labeled (express your answer in polar form).

(10) b. Calculate the total complex power supplied by the ideal three-phase source.



Note (if required):

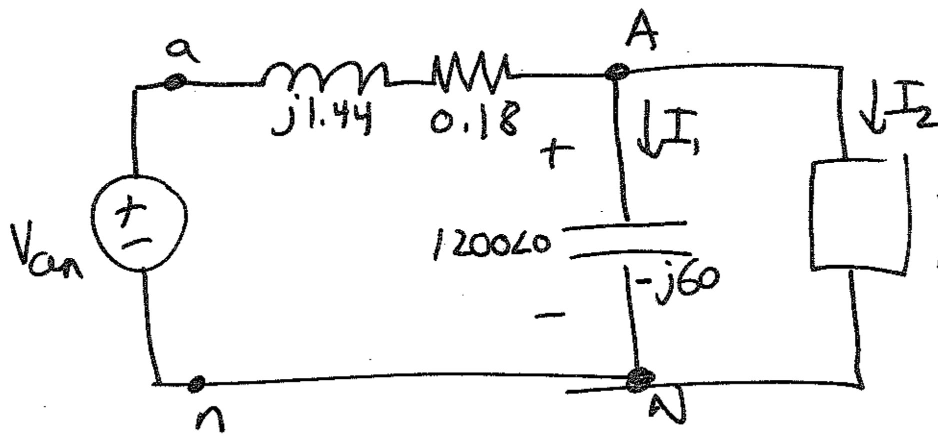


$|I_{line}| = \sqrt{3}\,|I_p|$

$\theta_{line} = \theta_p - 30°$

Line-phase current Relationship for Δ

a)



$S = V_{rms} I_{rms}^{*} = 166.67\angle 16.26$ KVA

$I_2 = \left(\frac{166.67k\angle 16.26}{1200\angle 0}\right)^{*}$

$= 133.3 - j38.89 \quad (138.8\angle -16.3)$

$I_1 = \frac{V_{AN}}{-j60}$

$= \frac{1200\angle 0}{-j60} = j20$

$I_{na} = I_1 + I_2 = 133.34 - j18.89$

$= 134.7\angle -8.06° A$

b) $S_{supplied} = S_{TL} + S_L + S_C$

$= I_{na}^2(0.18 + j1.44) + 166.67K\angle 16.26$

$+ -j\left(\frac{1200^2}{60}\right)$

$= (134.7)^2(0.18 + j1.44) + 166.67K\angle 16.26 - j24K$

$= 3265 + j26127.5 +$

$= 163270 + j48800$

$= 163.3 + j48.8 KVA$

$170.4 KVA\angle 16.6°$

$S_{Total} = 3 S_{supplied} = \boxed{511.2 KVA\angle 16.6°}$